The Calm Policymaker:

Imperfect Common Knowledge in New Keynesian Models

John Barrdear[1]

[1] Bank of England and Centre for Macroeconomics

June 2018

Disclaimer: The views expressed are those of the author and do not necessarily reflect the views of the Bank of England or its Committees.

How do monetary policy and nominal stability work?

- A long literature on price level determinacy.
- Does nominal stability require an ‘active’ central bank?
  - “Naïvely estimated” interest rate rules often violate the Taylor principle.
- Can we safely raise the inflation target?
- Is the liquidity trap real?
- Is neo-Fisherianism real?

What I do

- Explore determinacy in models with Imperfect Common Knowledge.
- Apply this to the New Keynesian model in particular.
- Four sources of indeterminacy in DSGE models:
  1. Multiple steady-state equilibria.
  2. For each steady state, multiple purely-forward-looking solutions.
  3. For each forward solution, multiple backward-looking solutions.
  4. For each backward solution, multiple rational bubbles.
- Three ingredients to ICK:
  - Agents are fully (Bayes) rational.
  - Agents lack full information. Each observes idiosyncratically noisy signals.
  - There is strategic interaction between agents.

Key results under ICK

- The NK model solves uniquely, regardless of the Taylor principle.
  - No backward-looking solutions or bubbles $\Rightarrow$ no need for Blanchard-Kahn.

- Standard results remain when the Taylor principle is satisfied.

- When $\phi_\pi < \phi_\pi^{\text{Taylor}}$, the price level – not just inflation – is stationary.
  - Persistence is a function of central bank design, increasing in both ϕ_π and ϕ_y.

- A unique and stable solution exists under an interest rate peg.
  - cf. Sargent & Wallace (1975).

Related literature

- **ICK/Dispersed information:** Woodford (2003); Nimark (2008, 2017); Lorenzoni (2009); Angeletos & La'O (2009, 2010, 2015); Graham & Wright (2010); Graham (2011); Melosi (2014); Kohlhas (2014); Angeletos & Lian (2016).
- **Equilibrium selection:** Blanchard & Kahn (1980) (+ Uhlig + Klein + Sims); Woodford (2001); Evans & Honkapohja (2003); many others.
- **Cochrane vs McCallum:** Cochrane (2007); McCallum (2009a); Cochrane (2009); McCallum (2009b); Cochrane (2011); McCallum (2012b).
- **The liquidity trap:** Benhabib, Schmitt-Grohé and Uribe (2001); many others.
- **The neo-Fisherian question:** Cochrane (2017); García-Schmidt & Woodford (2015).
- **Uniqueness in global games:** Morris & Shin (2000, 2002).

A toy model

A hidden state: $x_t = \rho x_{t-1} + u_t$ $\quad u_t \sim N\left(0, \sigma_u^2\right)$

Agent *i*'s signal: $s_{i,t} = x_t + v_{i,t}$ $\quad v_{i,t} \sim N\left(0, \sigma_v^2\right)$

A toy model

A hidden state: $x_t = \rho x_{t-1} + u_t$ $\qquad u_t \sim N\left(0, \sigma_u^2\right)$

Agent i's signal: $s_{i,t} = x_t + v_{i,t}$ $\qquad v_{i,t} \sim N\left(0, \sigma_v^2\right)$

Agent i's action: $z_{i,t} = E_{i,t}\left[x_t\right] + \beta E_{i,t}\left[z_{t+1}\right]$ $\qquad z_t \equiv \int_0^1 z_{i,t}\, di$

A toy model

A hidden state: $x_t = \rho x_{t-1} + u_t$ $\quad u_t \sim N\left(0, \sigma_u^2\right)$

Agent i's signal: $s_{i,t} = x_t + v_{i,t}$ $\quad v_{i,t} \sim N\left(0, \sigma_v^2\right)$

Agent i's action: $z_{i,t} = E_{i,t}\left[x_t\right] + \beta E_{i,t}\left[z_{t+1}\right]$ $\quad z_t \equiv \int_0^1 z_{i,t}\, di$

The average action: $z_t = \overline{E}_t\left[x_t\right] + \beta \overline{E}_t\left[z_{t+1}\right]$

Solving the baby model with full information

$$z_t = x_t + \beta E_t [z_{t+1}] \qquad x_t = \rho x_{t-1} + u_t$$

Solving the baby model with full information

$$z_t = x_t + \beta E_t [z_{t+1}] \qquad x_t = \rho x_{t-1} + u_t$$

The full set of solutions (following Blanchard, 1979):

$$z_t = (1 - \xi_t)\, z_t^{(F)} + \xi_t\, z_t^{(B)} + \theta_t \quad \text{where} \quad z_t^{(F)} = \sum_{q=0}^{\infty} (\beta\rho)^q x_t$$

$$\xi_t \in \mathbb{R} \qquad z_t^{(B)} = \frac{1}{\beta}(z_{t-1} - x_{t-1})$$

$$\theta_t = \beta E_t [\theta_{t+1}]$$

Solving the baby model with full information

$$z_t = x_t + \beta E_t \left[z_{t+1} \right] \qquad x_t = \rho x_{t-1} + u_t$$

The full set of solutions (following Blanchard, 1979):

$$z_t = (1 - \xi_t)\, z_t^{(F)} + \xi_t\, z_t^{(B)} + \theta_t \quad \text{where} \quad z_t^{(F)} = \sum_{q=0}^{\infty} (\beta\rho)^q x_t$$

$$\xi_t \in \mathbb{R} \qquad z_t^{(B)} = \frac{1}{\beta}\left(z_{t-1} - x_{t-1}\right)$$

$$\theta_t = \beta E_t \left[\theta_{t+1} \right]$$

The Blanchard-Kahn conditions:

A1: z_t is stationary

A2: $\left| \frac{1}{\beta} \right| > 1$

Assumption A2 makes both the backward solution and non-zero bubbles violate assumption A1.

$\Rightarrow$ Only $z_t = z_t^{(F)}$ remains: $\xi_t = \theta_t = 0 \ \forall t$

Solving the model with dispersed info

$$z_t = \overline{E}_t [x_t] + \beta \overline{E}_t [z_{t+1}] \qquad x_t = \rho x_{t-1} + u_t \qquad s_{i,t} = x_t + v_{i,t}$$

- The Kalman filter ensures that the hierarchy of expectations is AR(1):

$$\underset{(\infty \times 1)}{X_t \equiv \begin{bmatrix} x_t \\ \overline{E}_t [X_t] \end{bmatrix}} = \Phi X_{t-1} + \Psi u_t$$

Solving the model with dispersed info

$$z_t = \overline{E}_t [x_t] + \beta \overline{E}_t [z_{t+1}] \qquad x_t = \rho x_{t-1} + u_t \qquad s_{i,t} = x_t + v_{i,t}$$

- The Kalman filter ensures that the hierarchy of expectations is AR(1):

$$X_t \equiv \underset{(\infty \times 1)}{\begin{bmatrix} x_t \\ \overline{E}_t [X_t] \end{bmatrix}} = \Phi X_{t-1} + \Psi u_t$$

- Define S and T such that: $\begin{aligned} SX_t &= x_t \\ TX_t &= \overline{E}_t [X_t] \end{aligned}$

- The purely forward-looking solution is:

$$z_t = ST (I - \beta \Phi T)^{-1} X_t \quad \xrightarrow{\sigma_v \to 0} \left(\frac{1}{1 - \beta \rho} \right) x_t$$

Backward-looking solutions: example 1

$$z_t = \overline{E}_t [x_t] + \beta \overline{E}_t [z_{t+1}] \qquad x_t = \rho x_{t-1} + u_t \qquad s_{i,t} = x_t + v_{i,t}$$

Consider the equivalent to the purely backward solution under full info:

$$z_t = \tfrac{1}{\beta} \left(z_{t-1} - \overline{E}_{t-1} [x_{t-1}] \right) \quad \text{: Candidate solution}$$

Backward-looking solutions: example 1

$$z_t = \overline{E}_t\left[x_t\right] + \beta \overline{E}_t\left[z_{t+1}\right] \qquad x_t = \rho x_{t-1} + u_t \qquad s_{i,t} = x_t + v_{i,t}$$

Consider the equivalent to the purely backward solution under full info:

$$z_t = \tfrac{1}{\beta}\left(z_{t-1} - \overline{E}_{t-1}\left[x_{t-1}\right]\right) \qquad \text{: Candidate solution}$$

Step it forward and take the average expectation:

$$\overline{E}_t\left[z_{t+1}\right] = \frac{1}{\beta}\overline{E}_t\left[z_t - \overline{E}_t\left[x_t\right]\right]$$

Backward-looking solutions: example 1

$$z_t = \overline{E}_t\left[x_t\right] + \beta\overline{E}_t\left[z_{t+1}\right] \qquad x_t = \rho x_{t-1} + u_t \qquad s_{i,t} = x_t + v_{i,t}$$

Consider the equivalent to the purely backward solution under full info:

$$z_t = \tfrac{1}{\beta}\left(z_{t-1} - \overline{E}_{t-1}\left[x_{t-1}\right]\right) \qquad : \text{Candidate solution}$$

Step it forward and take the average expectation:

$$\overline{E}_t\left[z_{t+1}\right] = \frac{1}{\beta}\overline{E}_t\left[z_t - \overline{E}_t\left[x_t\right]\right]$$

Problems:

- Others' actions are unknown: $\overline{E}_t\left[z_t\right] \neq z_t$
- The LIE breaks down: $\overline{E}_t\left[\overline{E}_t\left[x_t\right]\right] \neq \overline{E}_t\left[x_t\right]$

Backward-looking solutions: example 2

Consider $$z_t = \delta' X_t + \phi z_{t-1}$$

Backward-looking solutions: example 2

Consider $$z_t = \delta' X_t + \phi\, z_{t-1}$$

This requires, *inter alia*, $\overline{E}_t\left[z_{t-1}\right] = \chi\, z_{t-1}$ where $\chi = \dfrac{1}{\beta \phi^2}$

Backward-looking solutions: example 2

Consider $z_t = \delta' X_t + \phi\, z_{t-1}$

This requires, *inter alia*, $\overline{E}_t\left[z_{t-1}\right] = \chi\, z_{t-1}$ where $\chi = \dfrac{1}{\beta\phi^2}$

But $\overline{E}_t\left[z_{t-1}\right] = \overline{E}_{t-1}\left[z_{t-1}\right] + h_t \underbrace{\left\{\rho x_{t-1} + u_t - \overline{E}_{t-1}\left[x_t\right]\right\}}_{\text{Average information obtained from agents' signals}}$

Backward-looking solutions: example 2

Consider $$z_t = \delta' X_t + \phi z_{t-1}$$

This requires, *inter alia*, $\overline{E}_t [z_{t-1}] = \chi z_{t-1}$ where $\chi = \dfrac{1}{\beta\phi^2}$

But $$\overline{E}_t [z_{t-1}] = \overline{E}_{t-1} [z_{t-1}] + h_t \underbrace{\left\{\rho x_{t-1} + u_t - \overline{E}_{t-1} [x_t]\right\}}_{\text{Average information obtained from agents' signals}}$$

Contradiction:

- z_{t-1} cannot be a function of u_t $\Rightarrow h_t = 0$
- $\rho > 0 \Rightarrow s_{i,t}$ is informative about X_{t-1} and z_{t-1} $\Rightarrow h_t \neq 0$

Rational bubbles: example

- Candidate solution: $z_t = \delta' X_t + \theta_t$ where $\theta = \beta E_t^{\Omega}[\theta_{t+1}]$
- A share, $\xi \in (0,1)$, of agents have ICK. The rest have full info:

$$s_{i,t}^{\theta} = \begin{cases} \theta_t + e_{i,t} \quad \text{where} \quad e_{i,t} \sim N\left(0, \sigma_e^2\right) & \text{if } i \in [0,\xi) \\ \theta_t & \text{if } i \in [\xi, 1] \end{cases}$$

Rational bubbles: example

- Candidate solution: $z_t = \delta' X_t + \theta_t$ where $\theta = \beta E_t^{\Omega}[\theta_{t+1}]$
- A share, $\xi \in (0,1)$, of agents have ICK. The rest have full info:

$$s_{i,t}^{\theta} = \begin{cases} \theta_t + e_{i,t} \quad \text{where} \quad e_{i,t} \sim N\left(0, \sigma_e^2\right) & \text{if } i \in [0, \xi) \\ \theta_t & \text{if } i \in [\xi, 1] \end{cases}$$

- Substitute the candidate into the equilibrium condition:

$$\begin{aligned} z_t &= \delta' X_t + \beta\left(\xi \overline{E}_t^{\theta}[\theta_{t+1}] + (1-\xi) E_t^{\Omega}[\theta_{t+1}]\right) \\ &= \delta' X_t + \theta_t + \xi\left(\overline{E}_t^{\theta}[\theta_t] - \theta_t\right) \end{aligned}$$

Rational bubbles: example

- Candidate solution: $z_t = \boldsymbol{\delta}' X_t + \theta_t$ where $\theta = \beta E_t^{\Omega}[\theta_{t+1}]$

- A share, $\xi \in (0,1)$, of agents have ICK. The rest have full info:

$$s_{i,t}^{\theta} = \begin{cases} \theta_t + e_{i,t} \quad \text{where} \quad e_{i,t} \sim N\left(0, \sigma_e^2\right) & \text{if } i \in [0, \xi) \\ \theta_t & \text{if } i \in [\xi, 1] \end{cases}$$

- Substitute the candidate into the equilibrium condition:

$$\begin{aligned} z_t &= \boldsymbol{\delta}' X_t + \beta \left(\xi \overline{E}_t^{\theta}[\theta_{t+1}] + (1 - \xi) E_t^{\Omega}[\theta_{t+1}] \right) \\ &= \boldsymbol{\delta}' X_t + \theta_t + \xi \left(\overline{E}_t^{\theta}[\theta_t] - \theta_t \right) \end{aligned}$$

- Contradiction:
  - Requires either $\xi = 0$ or $\sigma_e = 0$, both of which imply universal full information.

The end result

- When **any** positive mass of agents observe the state with **any** idiosyncratic noise, backward-looking solutions cannot exist.

- When **any** positive mass of agents observe bubbles with **any** idiosyncratic noise, rational bubbles cannot exist.

Intuition:

- Backward-looking solutions & rational bubbles require co-ordination.
- Co-ordination requires common knowledge.
- With idiosyncratic noise, common knowledge is absent.

Model outline

- Start from the canonical three-equation NK model with Calvo pricing.
- Log-linearise around a zero inflation trend.
- The representative household and central bank have full information.
- Price-setting firms are subject to imperfect common knowledge.

The HH and the CB

Standard Euler equation and Taylor-type rule:

$$y_t = E_t^{\Omega}[y_{t+1}] - \sigma\left(i_t - \left(E_t^{\Omega}[p_{t+1}] - p_t\right) - x_t\right)$$

$$i_t = \phi_y y_t + \phi_\pi (p_t - p_{t-1})$$

- $E_t^{\Omega}[\bullet] \equiv E[\bullet|\Omega_t]$ is the expectation under full information.
- x_t is a household preference shock (the natural rate of interest)

$$x_t = \rho x_{t-1} + u_t \qquad u_t \sim N\left(0, \sigma_u^2\right)$$

The Phillips Curve

- With Calvo pricing and dispersed information, the price level follows:

$$p_t = \theta p_{t-1} + (1 - \theta - \beta\theta)\,\overline{E}_t\left[p_t\right] + (\beta\theta)\,\overline{E}_t\left[p_{t+1}\right] + (1 - \theta)(1 - \beta\theta)\,\overline{E}_t\left[mc_t\right]$$

The Phillips Curve

- With Calvo pricing and dispersed information, the price level follows:

$$p_t = \theta p_{t-1} + (1-\theta-\beta\theta)\,\overline{E}_t\left[p_t\right] + (\beta\theta)\,\overline{E}_t\left[p_{t+1}\right] + (1-\theta)(1-\beta\theta)\,\overline{E}_t\left[mc_t\right]$$

- The Incomplete Information NKPC:

$$\begin{aligned}\pi_t = (1-\theta)\,\overline{E}_t\left[\pi_t\right] - (1-\theta)\left\{p_{t-1} - \overline{E}_t\left[p_{t-1}\right]\right\} \\ + (\beta\theta)\,\overline{E}_t\left[\pi_{t+1}\right] + (1-\theta)(1-\beta\theta)\,\overline{E}_t\left[mc_t\right]\end{aligned}$$

The Phillips Curve

- With Calvo pricing and dispersed information, the price level follows:

$$p_t = \theta p_{t-1} + (1-\theta-\beta\theta)\,\overline{E}_t\left[p_t\right] + (\beta\theta)\,\overline{E}_t\left[p_{t+1}\right] + (1-\theta)(1-\beta\theta)\,\overline{E}_t\left[mc_t\right]$$

- The Incomplete Information NKPC:

$$\begin{aligned}\pi_t = (1-\theta)\,\overline{E}_t\left[\pi_t\right] - (1-\theta)\left\{p_{t-1} - \overline{E}_t\left[p_{t-1}\right]\right\} \\ + (\beta\theta)\,\overline{E}_t\left[\pi_{t+1}\right] + (1-\theta)(1-\beta\theta)\,\overline{E}_t\left[mc_t\right]\end{aligned}$$

- Under full information, this is the canonical NKPC:

$$\pi_t = \beta E_t^{\Omega}\left[\pi_{t+1}\right] + \frac{(1-\theta)(1-\beta\theta)}{\theta} mc_t$$

Information

- The underlying state is the what the state would be under full info:

$$\boldsymbol{\eta}_t \equiv \begin{bmatrix} x_t \\ p_{t-1} \end{bmatrix} \quad \begin{array}{l} \text{: Today's demand shock} \\ \text{: Yesterday's price level} \end{array}$$

- Firms have dispersed information:

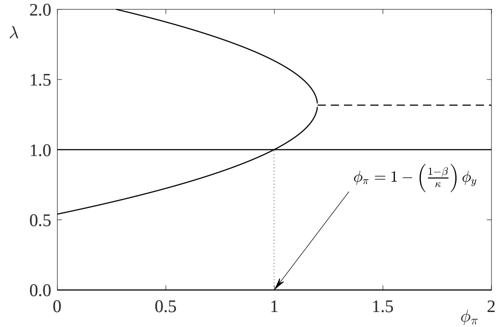
$$\mathcal{I}_{i,t} = \{\mathcal{I}_{i,t-1}, \boldsymbol{s}_{i,t}\}$$

$$\boldsymbol{s}_{i,t} = \boldsymbol{\eta}_t + \boldsymbol{v}_{i,t} \qquad \boldsymbol{v}_{i,t} \sim N(\mathbf{0}, \Sigma_v)$$

- This nests full information as a limiting case:

$$\boldsymbol{s}_{i,t}(i) = \begin{bmatrix} x_t & + & v_t^x(i) \\ p_{t-1} & + & v_t^p(i) \end{bmatrix} \xrightarrow{\Sigma_v \to 0} \text{Full information}$$

Eigenvalues of the NK model with full information

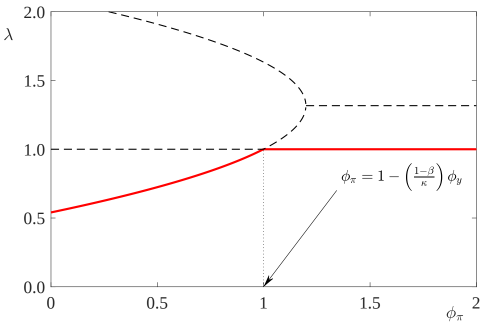


- $\phi_\pi > \phi_\pi^{\text{Taylor}}$: two eigenvalues outside the unit circle
- $\phi_\pi < \phi_\pi^{\text{Taylor}}$: only one eigenvalue outside the unit circle

The purely forward-looking solution under full info

Proposition 1

- The forward solution is found with forward substitution (Cho & Moreno, 2011).
- With distinct eigenvalues, this is the minimal solvent (Rendahl, 2017).



$$p_t = \lambda p_{t-1} + \gamma x_t$$

Solving the NK model with ICK

The purely forward-looking solution under full information:

$$p_t = \lambda p_{t-1} + \gamma x_t$$

Proposition 2

- The unique solution under ICK (θ is the Calvo parameter):

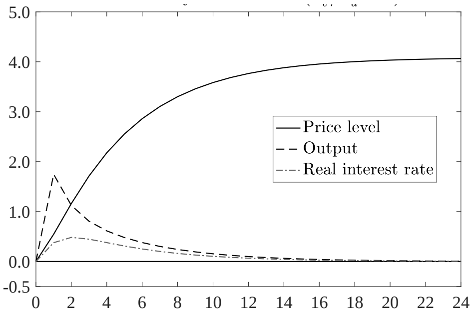
$$p_t = \theta p_{t-1} + (\lambda - \theta)\, \widetilde{p}_{t-1|t} + \gamma\, \widetilde{x}_{t|t}$$

- $\widetilde{x}_{t|t}$ and $\widetilde{p}_{t-1|t}$ are weighted averages of higher-order beliefs.
    - As in the toy model, the Kalman filter means that these follow a vector AR(1).
- The solution under ICK equals the purely-forward solution under full information when $\sigma_v = 0$ and approaches it smoothly as $\sigma_v \to 0$.

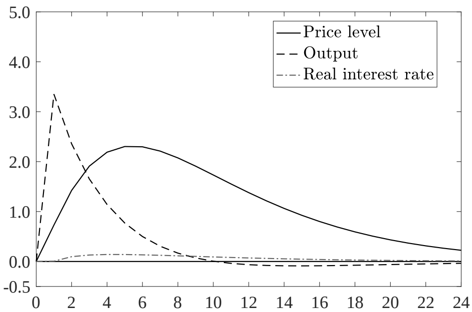
No need for B-K conditions $\Rightarrow$ the Taylor principle is not necessary.

Impulse Responses

$$i_t = 0.1\, y_t + 1.5\, \pi_t$$



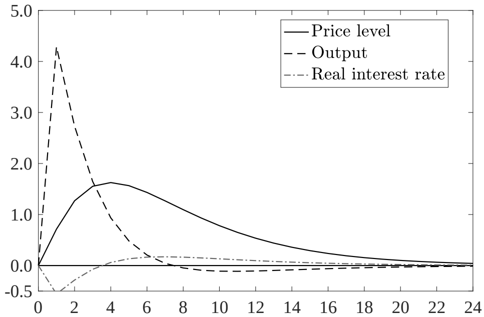
$$i_t = 0.1\, y_t + 0.5\, \pi_t$$



- The strong reaction of the nominal rate raises the real rate.
- As with price level targetting, future deflation raises the real rate.

Sargent & Wallace (1975) is not robust to ICK

Determinacy remains under an interest rate peg ($\phi_y = \phi_\pi = 0$):

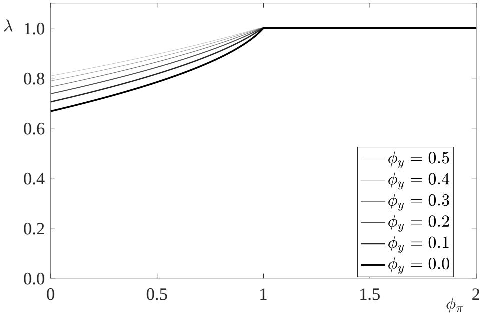


- This is pegged at the steady-state interest rate.
- A different peg would be a change *of* steady state.

What determines nominal persistence?

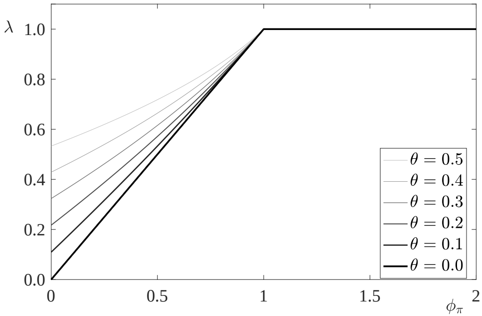
$$p_t = \theta p_{t-1} + (\lambda - \theta)\,\widetilde{p}_{t-1|t} + \gamma\,\widetilde{x}_{t|t}$$

Central bank design



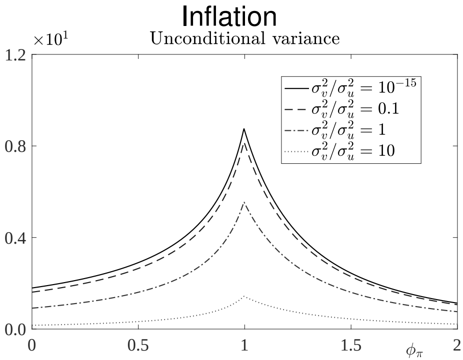
- λ is increasing in both ϕ_y and ϕ_π.

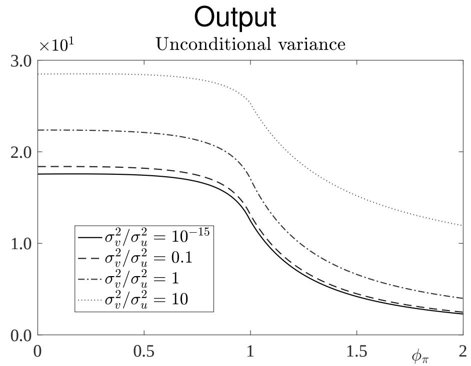
Price flexibility



- λ is increasing in θ.

Unconditional volatility



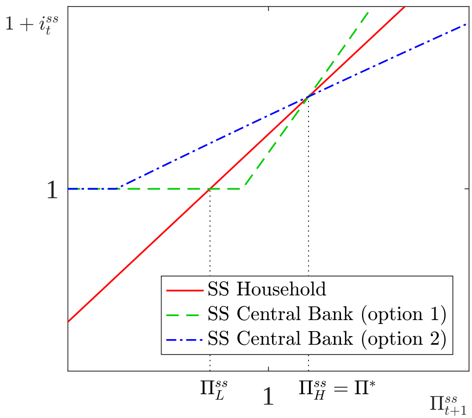


- Two channels:
  1. Var increases in λ and impact size
  2. Standard damping argument
- $\phi_\pi < \phi_\pi^{\text{Taylor}} \Rightarrow 1^{\text{st}}$ effect dominates.
- $\phi_\pi > \phi_\pi^{\text{Taylor}} \Rightarrow$ Only 2^{nd} effect varies.

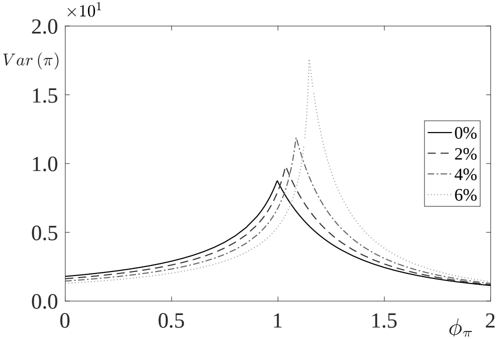
- Equivalent channels.
- But for $\phi_\pi < \phi_\pi^{\text{Taylor}}$, on-impact impulse gets much larger.

Other implications

No more liquidity trap?



Safe to raise the inflation target?



Conclusion

- The Taylor principle is not required in NK models with ICK.
  - With ICK, there exists a unique solution to rational, forward-looking, linear models that does not rely on the Blanchard-Kahn conditions.
  - Equilibrium "selection": no backward solns or bubbles, despite rationality.
- Policy can be calm: it doesn't *need* to intervene.
  - I'm linearising around a full-info trend, though.
  - $\Rightarrow$ Implicitly assumes that long-run expectations are well anchored.
- When $\phi_\pi < \phi_\pi^{\text{Taylor}}$:
  - The price level – not just inflation – is stationary.
  - Inflation volatility *falls*, but output volatility remains high.